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Section I – Kinesiology

Evaluation of the Final Time and Velocity of a 100 m Run Under the Realistic Conditions

by
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The aim of the research was to provide an analytical expression for the final time and velocity at the 100 m run, taking into account realistic conditions of the run, more precisely the effect of the wind and resistance of the medium (air). Combining the polynomial model for the distance vs time with the solution of the algebraic cubic equation, such an analytical expression was derived. The expression allowed to evaluate the dependence of the final time of the race on the wind velocity. This enabled the quantification of the time effect of the mentioned influences on the final time and velocity. It is possible to calculate the dependence of the sprinter's velocity on expired running time for various wind velocities (from 0 up to ± 10 m/s) as well as determine the maximal running velocity v_{max} and corresponding time moment t_{max}. The results obtained were verified using split time data for six top sprinters: C. Lewis, M. Green, U. Bolt and F. Griffith-Joyner, E. Ashford and H. Drechsler. The results confirmed that it was possible to quantify the time effect of the influence of the wind velocity and resistance of the medium on the final time of the 100 m run. Although the applicability of the approach was tested using the data concerning top sprinters, the mathematical expressions involved are simple enough to be used by any coach to estimate the results of a sprinter under various weather conditions.

Key words: *100 m sprint, final time, wind velocity influence, polynomial model.*

Introduction

This research was initiated in order to obtain an analytical expression providing a numerical quantification of the time effect of the influence of the wind and resistance of the medium on the final time of the 100 m run, although the International Association of Athletic Federations (IAAF) is not officially recognizing such quantification yet. The interest for such mathematical models dates back to the beginning of the 20th century. It begins with the pioneering works of Meade (1916), Hill (1925, 1928), after whom there comes Keller (1974) who used a differential equation to describe a model for determining the velocity during the run, under the assumption of the constant muscle force in a 100 m sprint. Later, the model was extended to include the effect of the wind (Dapena and Feltner, 1987; Ward-Smith, 1985, 1999; Prichard, 1993; Behncke, 1994). In addition, Linthorne (1994) performed a comprehensive statistical analysis of the wind effect on 100 m time performance. Mureika (2001, 2003) presented a quasi-physical model based on a system of coupled differential equations with modification for drag effect, wind effect and velocity included. In our approach, we used the polynomial model for the distance *vs* time combined with the solutions of the algebraic cubic equation.

Time effect of the wind influence implies that the wind blowing during the race into sprinters' back (tail wind) contributes to a decrease, and in the case of a frontal blow (head wind), to an increase of the final time t_f of the run. The increase of the final time for the head wind is

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larger than the corresponding decrease of the final time in the case of the tail wind.

In order to quantify the time effect of the different wind velocities on final time of the run of a particular sprinter, let us hypothetically assume that the run occurs under the condition of zero wind velocity: $w=0$. During the run with $w=0$, the velocity of the sprinter with respect to the ground is the same as the velocity with respect to the air (medium). In such a case the final time equals t_0. In the case of wind presence ($w \neq 0$), it equals t_f which is lower than t_0 for $+w$ (tail wind – common convention) and higher for $-w$ (head wind). The time effect Δt affecting the final time is defined as $\Delta t = t_0 - t_f$, with $\Delta t \geq 0$ for the tail wind, and $\Delta t \leq 0$ for the head one. The data on the final time and velocity of the run, together with the wind velocity assumed to be constant during the run can be found in the official reports of the IAAF. (We are aware of the fact that the assumption of the constancy of wind velocity, both in the value and in direction is just an approximation justified by the idea that at such short distances, the variations of the wind velocity can be neglected.)

IAAF recognizes the final time as a record if it is achieved with the tail wind velocities up to 2 m/s, although, in the work of Janjić et al. (2017) it was demonstrated by the plausible results that the limit for w should be 1 m/s,

Obviously, one needs a numerical relationship between the above mentioned quantities. The aim of this paper is to offer a mathematically correct and plausible answer. Another argument in favour of producing quantified value for the time effect Δt is that although its values often seem negligible, they actually are not, but must be used as the correction term for the final time t_f.

Since the same final time can be achieved by different sprinters under various wind velocities and resistance of the medium, it turns out that the final time is not always the reliable single factor of the placement. This is confirmed by the following three examples (IAAF, 2017) which unambiguously indicate that the ranking list demands a correction of the measured final time with quantified time effect of the wind influence.

Bolt's 100 m run in Bruxelles (2008) that ended with the recorded final time t_f = 9.77 s and with the head wind velocity w = 1.3 m/s was ranked at the 9th place in the world ranking. If we perform the correction of the final time using the time effect, which according to our calculation rounded to two decimal digits equals 0.02 s, new corrected final time of the run would be t_f = 9.77-0.02 = 9.75 s. The corrected time result would move Bolt from the 9th to the 5th place, 4 places ahead.

Tyson Gay ran 100 m in Shanghai in 2008 in a final time of t_f = 9.69 s with tail wind velocity w = +2 *m/s*. He was ranked in the second place of the world list. If, however, the correction of the final time had rounded to two decimal digits due to the estimated time effect, the new corrected final time of the run would have been t_f = 9.69 + 0.04 = 9.73 s. With this correction, Gay would have moved from the 2nd to the 4th place of the world ranking.

Asafa Powell achieved in 2007 in Rieta, with no wind (w = 0 m/s), a 100 m final time of t_f = 9.78 s. Powell was 16th on the ranking list. If there had been a correction for the influence of wind velocity and resistance of the medium, Powell would have been ranked 9th, since sprinters classified at higher positions in the ranking list were running in the presence of a tail wind.

Three aforementioned examples, chosen among many others, indicate that the single factor deciding about the placement of the sprinter in the ranking list was the recorded final time of the run.

For the sake of testing the theory and demonstrating the necessity of the quantification of time effect of the influence of the wind velocity and resistance of the medium on the final time, we shall apply our analysis to the results of top sprinters – male: Lewis, Green and Bolt, and female: Griffith-J, Ashford, Drechsler.

Methods

The present work was conformed with the Ethical Code of the Research of the University of Novi Sad.

In order to quantify the time effect of the influence of the wind velocity and the resistance of the medium on the final time in the 100 m run ("time effect", further on) we started from the general solution of the following differential equation:

$$m\frac{dv}{dt} = F(t)\text{-}kv \pm lw^2 \qquad (1)$$

with: m – sprinter's mass, $F(t)= F$ – constant force of the sprinter (Doder et al., 2012; Janjić et al., 2016), kv – force of the resistance of the medium, k – coefficient of the resistance of the medium (air), lw^2–force of the wind influence, w – constant wind velocity colinear with sprinter velocity, l – coefficient of the resistance of the medium with wind, which according to common practice we took equal for all sprinters l = 0.3 kg/m (Helene and Yakachita, 2010).

The solution of the equation (1) (Keller, 1974) with the initial condition for $t = 0$, $v = v_0$, is the function:

$$v = v_0 + \left(\frac{F \pm lw^2}{k} - v_0\right)\left(1 - e^{-\frac{k}{m}t}\right) \quad (2)$$

The distance covered in time t is obtained by integration of (2), $S = \int v dt$ with initial condition $S = 0$, $t = 0$

$$S = \frac{F \pm lw^2}{k} t - \frac{m}{k}\left(\frac{F \pm lw^2}{k} - v_0\right)\left(1 - e^{-\frac{k}{m}t}\right) \quad (3)$$

According to (2), the velocity has two limiting values: for $t = 0$ and $t \to \infty$, and no maximum, which contradicts the measurement data, so the expression (2) must be adapted to conform with this fact. This is achieved by expanding the expression $e^{-\frac{k}{m}t}$ into Taylor' series:

$$e^{-\frac{k}{m}t} = 1 - \left(\frac{k}{m}\right) t + \frac{1}{2!}\left(\frac{k}{m}\right)^2 t^2 - \frac{1}{3!}\left(\frac{k}{m}\right)^3 t^3$$

so that the velocity is expanded up to the square of time and the distance up to the third power (Janjić et al., 2014, 2016, 2017). Substituting this expansion into the expressions (2) and (3), we obtain:

$$v = v_0 + \frac{k}{m}\left(\frac{F \pm lw^2}{k} - v_0\right) t - \frac{1}{2}\left(\frac{k}{m}\right)^2 \left(\frac{F \pm lw^2}{k} - v_0\right) t^2 \quad (4)$$

$$S = v_o t + \frac{1}{2}\left(\frac{k}{m}\right)\left(\frac{F \pm lw^2}{k} - v_0\right) t^2 - \frac{1}{3!}\left(\frac{k}{m}\right)^2 \left(\frac{F \pm lw^2}{k} - v_0\right) t^3 \quad (5)$$

Introducing three positive coefficients P_1, P_2 and P_3 in the following way:

$$P_1 = v_o, \;\; P_2 = \frac{1}{2}\left(\frac{k}{m}\right)\left(\frac{F \pm lw^2}{k} - v_0\right) \;\; P_3 = \frac{1}{3!}\left(\frac{k}{m}\right)^2 \left(\frac{F \pm lw^2}{k} - v_0\right) \quad (6)$$

the expression (5) can be represented as a third order polynomial expression with no constant term, i.e.

$$S = P_1 t + P_2 t^2 - P_3 t^3 \quad (7)$$

One can notice from (6) that the coefficient P_2 depends on the wind velocity w and can be rewritten as:

$$P_2 = P_2^{(0)} \pm \frac{lw^2}{2m} \quad P_2^{(0)} = \frac{F - kv_0}{2m} \quad (8)$$

The coefficient $P_2^{(0)}$ describes the wind–free (w = 0) situation.

The expression (8) relates the results obtained with and without the wind allowing us to use the results obtained from the fit for one wind velocity to predict the results that would be obtained under the conditions of another wind velocity. As for the coefficient P_3, there exists one important relation according to (6), the ratio P_3 / P_2:

$$\frac{P_3}{P_2} = \frac{1}{3}\frac{k}{m} \quad i.e \quad P_3 = \frac{1}{3}\frac{k}{m} P_2 \quad and \quad k = 3m\frac{P_3}{P_2} \quad (9)$$

independent of the wind velocity. If we know the sprinter's mass, it allows the evaluation of k.

Using the least square fit for the equation (7) with measured segment data for S and t for a particular run, it is possible to obtain an analytical expression for $S = f(t)$. In this study we used the program *Mathematica 9*. The coefficients accompanying t, t^2 and t^3, provide numerical values of the coefficients P_1, P_2 and P_3, respectively.

Equation (7) (for fixed value of S = 100 m) is the algebraic cubic equation (for the final time of the run) with real coefficients. There may exist either three real roots (solutions) or a single real and two mutually complex conjugated ones. However, if we know in advance that all the roots are real, finding the solutions is simplified (Tignol, 2001). One way of knowing it is to represent graphically the right hand side of the equation (7) and look for the intersections of the curve with the horizontal line corresponding to S = 100 m. It can be seen that there are three intersections corresponding to real roots. One root corresponds to the negative time region, while the other two correspond to the positive one. The positive one with the lower value can be assigned to the final time of the run.

Once we know that the roots of the cubic equation are real, we can apply the so called Tschirnhaus–Vieta approach (Tignol, 2001) to obtain all three real roots, and it can be shown by the direct substitution of the measured values that out of the three possible solutions for time, only one corresponds to the realistic situation.

In this way we obtain the expression for the final time with explicit dependence on the wind velocity included. The coefficients k and $P_2^{(o)}$ describe the dependence on the resistance of the medium.

$$t_f = \frac{m}{k}\left\{1 + 2\sqrt{1 + \frac{k}{m}\frac{v_0}{P_2^{(0)} \pm \frac{lw^2}{2m}}}\cos\left[\frac{4\pi}{3} + \frac{1}{3}\arc\cos\frac{1+\frac{3k}{2m}\left(1-\frac{kS}{mv_0}\right)\frac{v_0}{P_2^{(0)} \pm \frac{lw^2}{2m}}}{\left(1+\frac{k}{m}\frac{v_0}{P_2^{(0)} \pm \frac{lw^2}{2m}}\right)^{3/2}}\right]\right\} \quad (10)$$

It is also convenient to reformulate the expression (4) for the sprinter's velocity v.

$$v = v_0 + 2P_2\, t - 3P_3\, t^2 = v_0 + P_2^{(0)}\left(2t - \frac{k}{m}t^2\right) \pm \frac{lw^2}{2m}\left(2t - \frac{k}{m}t^2\right) \quad (11)$$

One can notice that equation (11) includes the polynomial coefficient P_2 already expressed as the sum of the term independent of wind velocity $P_2^{(o)}$ and the term depending on the wind velocity. The following procedure is proposed. One starts from the value of the coefficient P_2 obtained from the fit to data measured under the conditions of the particular wind velocity w_1. Then, using (8), *i.e.* subtracting the $lw_1^2/2m$ term, one evaluates $P_2^{(o)}$. Now, substituting it back into (8), one can obtain P_2 for any wind velocity. With these coefficients, one can predict the value of the final time for any wind velocity w, i.e. derive the correction term. This is the basic advantage of using the polynomial model for the description of sprinting velocity and final time of the 100 m run.

Results

In order to test the above presented theory, we started from the results achieved by the top male sprinters, i.e. Lewis, Green and Bolt, and top female sprinters: Griffith-J, Ashford and Dechsler. Values for t and S in 10 m segments during a 100 m run for these sprinters according to the IAAF data are provided in Table 1. We also supplied the mass of each sprinter and the wind velocity for the particular run.

Using these data for t and S, least squares fit of the polynomial expression (7) was performed using program *Mathematica 9*. Fit produced the polynomial coefficients P_1, P_2 and P_3, respectively, defined by the expression (6); please notice that $P_1 = v_0$. Explicit values for the coefficients P_1, P_2 and P_3 with corresponding statistical data for each particular sprinter, rounded to three decimal digits are presented in Table 2 together with the values of necessary variables $P_2^{(0)}$ and k.

We would like to mention that the values of the coefficients are slightly different from the ones appearing in our previous works (Janjić et al., 2014, 2016, 2017) due to different accuracy of the present fitting with respect to the one used for fitting in previous papers. However, all the conclusions are independent of these differences.

Starting from expression (10) which is the analytical form of the final time t_f and data from Table 2, following the above described procedure we calculated the hypothetical final time for the 100 m run with no wind $w = 0$, and for various values of both tail ($+w$) and head ($-w$) wind in the range ±1 – ±10 m/s. The difference between values of the final times t_o ($w = 0$) and t_f for varying wind velocities ($+w$ and $-w$ in the given range) provides quantified values of the time effect Δt of the wind and the resistance of the medium influence on the final time of the run. The results obtained for both male and female sprinters are presented in Table 3 and we can clearly see that there appears a quantified time effect giving plausible results.

Another justification of the correctness of the presented analysis can be seen from the plot of the final time of the 100 m run *vs* wind velocity (Figure 1), for the particular case of Bolt. The curves are plotted as the analytical functions given by the equation (10).

The shape of the curve in Figure 1a is a convex one showing that with an increase of the tail wind velocity $+w$, final time decreases as expected, implying the increase of sprinting velocity. The concave shape of the curve (Figure 1b) indicates that the increase of the absolute value of the head wind $-w$ increases the final time while decreasing sprinting velocity.

Figure 2 shows the plots of the dependence of sprinting velocity v of Bolt in the time t, for various wind velocities in the range 0, ±1…±10 m/s. The curves do not intersect, yet they become shorter (shorter running time) with increasing maximal velocity so that the area under the curve is preserved. (One test of the correctness of the procedure is the integration, i.e. the calculation of the area below the curves, which always equals 100 m). Their maxima occur for the same time t_{max}, while the values of maximal velocity v_{max} for $+w$ increase along with wind velocity and the opposite behavior occurs for the head wind $-w$. The fact that the moment of achieving maximal velocity is independent of wind velocity is a peculiarity of our polynomial model. Since the velocity is a quadratic function of time (11), it is easy to derive the time for which the velocity reaches its maximum $t_{max} = P_2/3P_3$, in our particular case equal to $t_{max} = m/k$. Substituting t_{max} into (11), we see that the maximal sprinting velocity follows a simple quadratic dependence on wind velocity :

$$v_{max} = (v_0 + m/k\, P_2^{(0)}) \pm \frac{lw^2}{2k} \quad (12)$$

Table 1

Segment times for selected six top sprinters in the form (t[s], s[m])

Male:
C. Lewis:{(0,0), (1.89,10), (2.96,20), (3.90,30), (4.79,40), (5.65,50), (6.48,60), (7.33,70), (8.18,80), (9.04,90), (9.92,100)} (*m* = 81 kg, *w* = +1.1 m/s)
M. Green:{(0.0), (1.83,10), (2.83,20), (3.75,30), (4.64,40), (5.50,50), (6.33,60), (7.16,70), (8.02,80), (8.9,90), (9.22,100)} (*m* = 77 kg, *w* = +0.9 m/s)
U. Bolt:{(0.0), (1.89,10), (2.88,20), (3.78,30), (4.64,40), (5.47,50), (6.29,60), (7.01,70), (7.92,80), (8.75,90), (9.58,100)} (*m* = 86 kg, *w* = +0.9 m/s)
Female:
F. Griffith-Joyner:{(0.0), (2.10,10), (3.09,20), (4.09,30), (5.04,40), (5.97,50), (6.89,60), (7.8,70), (8.7,80), (9.62,90), (10.54,100)} (*m* = 59 kg, *w* = +3.0 m/s)
E. Ashford:{(0.0), (2.02,10), (3.13,20), (4.15,30), (5.11,40), (6.07,50), (7.01,60), (7.96,70), (8.91,80), (9.87,90), (10.83,100)} (*m* = 52 kg, *w* = +3.0 m/s)
H. Drechsler:{(0.0), (2.01,10), (3.12,20), (4.14,30), (5.11,40), (6.08,50), (7.02,60), (7.87,70), (8.92,80), (9.88,90), (10.86,100)} (*m* = 61 kg, *w* = +3.0 m/s)

Table 2

Polynomial coefficients with statistical data

Sprinter	Coef.	Estimate	St. error	t Statistic	*p*	R-Value
C. Lewis	P_1	3.448	0.236	14.608	$4.731 \cdot 10^{-7}$	0.999
	P_2	1.325	0.069	18.977	$6.152 \cdot 10^{-8}$	
	P_3	0.066	0.005	13.490	$8.745 \cdot 10^{-7}$	
M. Green	P_1	3.813	0.273	13.952	$5.748 \cdot 10^{-7}$	0.999
	P_2	1.349	0.082	16.475	$1.859 \cdot 10^{-7}$	
	P_3	0.072	0.006	12.246	$1.836 \cdot 10^{-6}$	
U. Bolt	P_1	3.422	0.323	10.607	$5.459 \cdot 10^{-6}$	0.9999
	P_2	1.442	0.099	14.589	$4.778 \cdot 10^{-7}$	
	P_3	0.075	0.007	10.321	$6.702 \cdot 10^{-6}$	
F. Griffith-J.	P_1	3.604	0.307	11.754	$2.5108 \cdot 10^{-6}$	0.999
	P_2	1.094	0.085	12.790	$1.3170 \cdot 10$	
	P_3	0.051	0.006	8.995	$0.000 \cdot 10$	
E. Ashford	P_1	3.637	0.315	11.552	$2.863 \cdot 10^{-6}$	0.998
	P_2	1.046	0.086	12.209	$1.879 \cdot 10^{-6}$	
	P_3	0.049	0.005	8.8667	$0.000 \cdot 10$	
H. Drechsler	P_1	3.558	0.308	11.563	$2.842 \cdot 10^{-6}$	0.999
	P_2	1.079	0.083	12.933	$1.210 \cdot 10^{-6}$	
	P_3	0.052	0.005	9.6128	$0.000 \cdot 10$	

Variables featuring in the equations (15,17 and 18)

	C. Lewis	M. Green	U. Bolt	F. Griffith-J.	E. Ashford	H. Drechsler
$P_2^{(0)}$	1.326	1.348	1.441	1.071	1.012	1.057
k[kg/s]	12.208	12.290	13.358	8.293	7.354	8.802

Note: $P_1 = v_0$

Table 3

Time effect Δt and the final time t_f as the function of the tail (+w) and head (–w) wind velocity for male and female sprinters.

+w		C. Levis		M. Grin		U. Bolt		F. Griffith-J.		E. Ashford		H. Drechsler	
		$t_i[s]$	$\Delta t = t_0 - t_i^*$	$t_i[s]$	$\Delta t = t_0 - t_i^*$	$t_i[s]$	$\Delta t = t_0 - t_i^*$	$t_i[s]$	$\Delta t = t_0 - t_i^*$	$t_i[s]$	$\Delta t = t_0 - t_i^*$	$t_i[s]$	$\Delta t = t_0 - t_i^*$
0	t_0	9.971		9.869		9.634		10.730		11.067		11.074	
1	t_1	9.962	9.10^{-3}	9.860	$9x10^{-3}$	9.626	8.10^{-3}	10.715	0.015	11.047	0.020	11.057	0.017
2	t_2	9.934	0.036	9.832	0.037	9.603	0.031	10.668	0.062	10.989	0.078	11.007	0.067
3	t_3	9.889	0.081	9.785	0.084	9.565	0.069	10.592	0.138	10.893	0.174	10.924	0.150
4	t_4	9.827	0.143	9.721	0.148	9.512	0.122	10.488	0.242	10.764	0.303	10.812	0.262
5	t_5	9.749	0.221	9.641	0.229	9.446	0.188	10.359	0.371	10.606	0.461	10.674	0.400
6	t_6	9.657	0.314	9.56	0.324	9.366	0.268	10.208	0.522	10.422	0.645	10.512	0.556
7	t_7	9.551	0.419	9.437	0.432	9.276	0.358	10.238	0.692	10.217	0.850	10.331	0.743
8	t_8	9.434	0.536	9.317	0.553	9.174	0.460	9.853	0.877	9.996	1.071	10.134	0.940
9	t_9	9.307	0.664	9.186	0.683	9.064	0.570	9.656	1.074	9.762	1.305	9.925	1.149
10	t_{10}	9.170	0.80	9.047	0.823	9.445	0.689	9.449	1.281	9.521	1.546	9.707	1.367
-w													
0	t_0	9.971		9.869		9.634		11.067		11.067		11.074	
-1	t_1	9.980	$-9 \cdot 10^{-3}$	9.879	-0.01^{-3}	9.642	$-8 \cdot 10^{-3}$	11.087	-0.020	11.087	-0.020	11.091	-0.017
-2	t_2	10.008	-0.037	9.908	-0.038	9.665	-0.031	11.147	-0.08	11.147	-0.08	11.143	-0.069
-3	t_3	10.054	-0.084	9.957	-0.087	9.705	-0.071	11.249	-0.182	11.249	-0.182	11.231	-0.157
-4	t_4	10.121	-0.151	10.026	-0.156	9.762	-0.128	11.399	-0.332	11.399	-0.332	11.359	-0.285
-5	t_5	10.209	-0.239	10.118	-0.248	9.836	-0.202	11.601	-0.534	11.601	-0.534	11.531	-0.457
-6	t_6	10.321	-0.350	10.234	-0.365	9.929	-0.295	11.867	-0.80	11.867	-0.80	11.755	-0.681
-7	t_7	10.548	-0.487	10.378	-0.509	10.004	-0.410	12.209	-1.142	12.209	-1.142	12.041	-0.967
-8	t_8	10.624	-0.654	10.554	-0.683	10.181	-0.547	12.655	-1.588	12.655	-1.588	12.406	-1.332
-9	t_9	10.825	-0.854	10.766	-0.897	10.346	-0.712	13.247	-2.180	13.247	-2.180	12.881	-1.807
-10	t_{10}	11.065	-1.094	11.025	-1.155	10.542	-0.908	14.087	-3.020	14.087	-3.020	13.527	-2.453

*i=1,2,3,…. 10

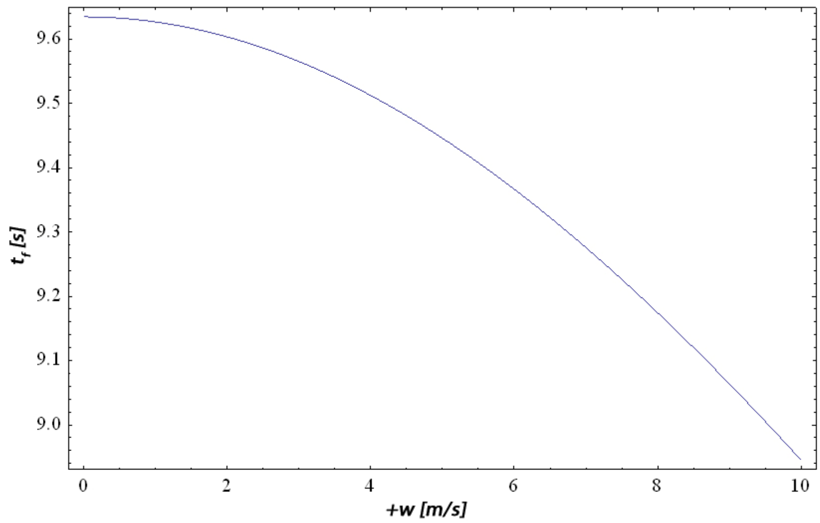

Figure 1a

Final time t_f dependence on the tail wind velocity +w for U. Bolt

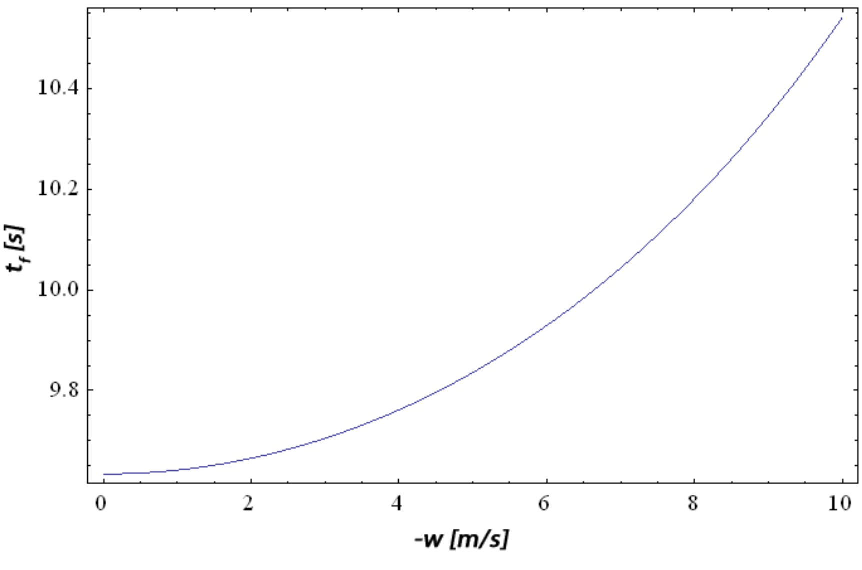

Figure 1b

Final time t_f dependence on the head wind velocity -w for U. Bolt

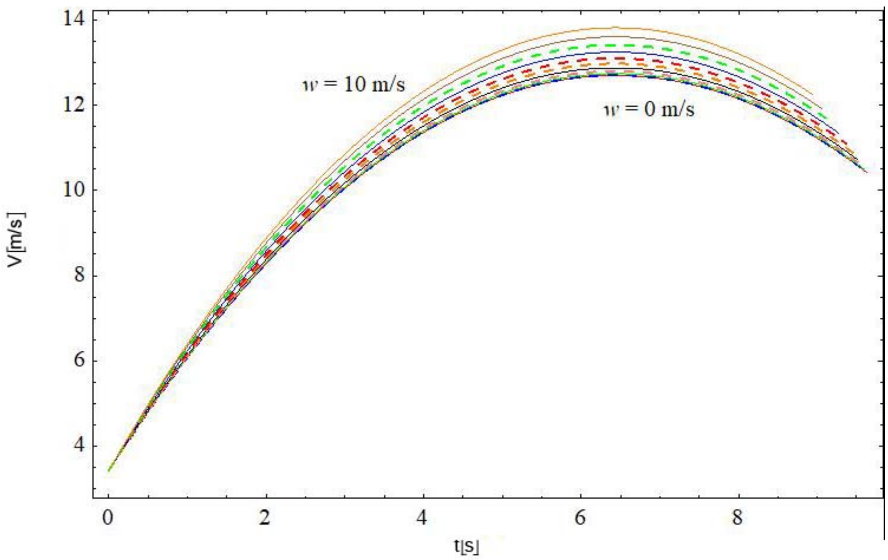

Figure 2a

The dependence of the sprinter velocity v on time t for various tail wind velocities + w (0, 1, 10 m/s) for U. Bolt

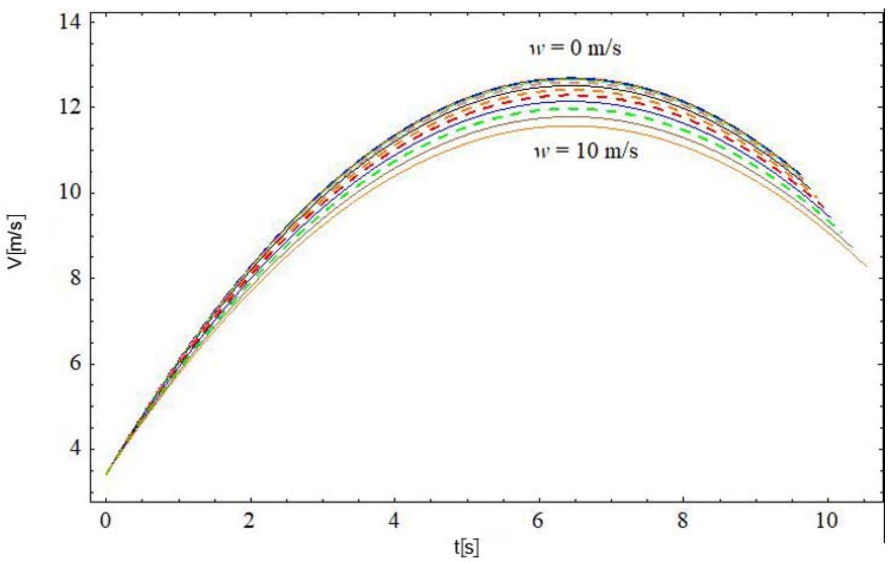

Figure 2b

The dependence of the running velocity v on time t for various head wind velocities – w (0, 1, …10 m/s) for U. Bolt

Discussion

The question of the quantification of the effect of the wind velocity and the resistance of the medium on the sprinter's velocity and final time of the 100 m run is an important one, especially in sprinting and other athletic disciplines, where the world ranking depends only on the result measured without any correction as the final time reached during a particular competition. Since the same final time can be achieved by different sprinters under various wind velocities and resistance of the medium, it turns out that the final time is not always the reliable single factor of the ranking. This is illustrated by the three examples discussed in the Introduction section of the paper.

These examples indicate that the single factor deciding about the ranking of the sprinter was the recorded final time of the run. Therefore, we would like to draw the attention of IAAF authorities, as well as other national athletic associations to the evident problem of non-existence of the corrected ranking list of final times, a problem to be considered in order to enable the objective evaluation of the achievements in athletics. In our opinion, it is sufficient to apply the above described procedure using a simple algebraic expression to calculate the theoretical final time t_f depending on various wind velocities $\pm w$ and resistance of the medium at the 100 m run.

A discussion between experts has been carried out about the effect of the wind velocity and resistance of the medium on the final time of the 100 m run and besides an intuitive estimate that the tail wind up to 2 m/s improves the result, various numerical estimates of the time effect appear: 0.09-0.14 s (Mureika, 2003), 0.19 s (Spiegel and Mureika, 2003) or on average about 0.1 s. However, our results of the time effect for male sprinters (Table 3) give much lower values for $+w$ up to 2 m/s. From our point of view, these lower values are realistic, since the wind up to 2 m/s (7.2 km/h) according to the Beaufort scale corresponds to the breeze which is comparable with the pedestrian velocity (1.6 m/s, 6 km/h) or velocity of a trained mountaineer (2.8 m/s, 10 km/h). It indicates that this wind exists, but it is energetically insufficient for a substantial influence on the final time. Yet we encounter a completely different situation for female sprinters (Table 3) whose run occurred with the wind velocity of 3 m/s (10.8 km/h). The results were not accepted for record purposes ($w > 2$ m/s), and it is obvious that a much stronger effect of wind velocity on the final time occurred. These two examples lead to the conclusion that the estimate of the improvement of the final time for wind velocity to 2 m/s of 0.1 s is too large and corresponds better to the wind velocities higher than 2 m/s.

This conclusion is supported by the results concerning both the effect of the tail wind velocity on increasing the final sprinter's velocity (as calculated from (12)) and quantified time effect on decreasing the final time of the 100 m run (Table 3). According to these results, the relative effect of the tail wind with velocity $w = +1$ m/s on increasing the final sprinter's velocity and thus decreasing the final time of the 100 m run for all studied sprinters is of the same order of magnitude of 10^{-4}. This almost equals the wind effect, does not introduce any advantage for the wind velocity of + 1 m/s, neither by an increase in the final velocity, nor by a decrease in the final time. However, for the wind velocity of +2 m/s, the relative effects are in the range of 10^{-4} - 10^{-3} so they are less suitable for the allowed boundary of the wind velocity than $w = +1$ m/s.

Looking at the top sprinters' results, in the 100 m run (probably in other events as well) the tail wind up to +1 m/s produces approximately equal small relative effect both in increasing the final velocity and in decreasing the final time of the run. This is why, in our opinion, the IAAF should accept the results for the records only if achieved with the tail wind velocity up to +1 m/s, since these are results reached by athletes without practically any external influence. It is best confirmed by the fact that U. Bolt under the wind velocity of +0.9 m/s achieved the world record in the 100 m run with the final time of 9.58 s. The effect of wind velocity of 0.9 m/s is energetically negligible, so it is the result of a superior athlete. If the limit remains +2 m/s, then a correction of the final time should be performed in the manner similar to the one described in our work.

Finally we would like to comment on the discrepancies between our results for the final time with respect to the measured ones which are within the error margins and results from the least squares fitting method applied. It minimizes the sum of the squares of the deviations between the measured values and fits the curve at all points while it is known that very often the final points are poorly reproduced. However, the methods which fit boundary points correctly do not reproduce the middle part of the curve properly, so they would give us poor values for the velocities.

Conclusions

The aim of this study was to offer a simple mathematical expression allowing to evaluate the time effect, i.e. the dependence of sprint time on wind velocity and air resistance. In this way, not only the results at competition can be treated in a different manner (as demonstrated using the results of the top sprinters), but also any coach can analyze this effect during training. Since the approach is applicable in various ways, and with improving mathematical software, it can be the subject of wide use in athletics.

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