DETERMINATION OF OPTIMAL CONTROLS OF ENDURANCE FORMATION IN BEGINNING SWIMMERS

by

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Value of each state variable (physical condition of the swimmer, sports results) at the end of a certain unit of time is a function of general condition of a swimmer, his results, described by state variables at the beginning of this time unit and of accepted training procedure, that is of realised intensity of individual training means (control variables) in the analysed unit of time [4]. It can be written symbolically in the following way:

$X(l+1) = X(l) + \Delta X(l)$ where $\Delta X(l) = \Phi(X(l), U(l))$; l means the number of training period, l=0, ..., N-1.

The aim of optimization of swimming training is to determine such a sequence of controls U(0), ..., U(N-1). that value of sports results at the end of the whole training period reaches its extremum $X(N) = \max(\min)$.

In order to use optimization methods and to automate development of a model, function Φ should be written with as simple formula as possible. The following form is suggested: $\Phi(X,U) = aX + cU + bUX + d$.

In the above formula, on the right side the first element (aX) can be interpreted as a description of influence of condition of the swimmer on changes of his results, the second one (cU) as influence of training and the third (bUX) as influence of combination of both condition and training. In order to use methods of determining optimal control in accordance with Pontriagin's principle the model should be sequenced, and increments in training periods replaced with derivatives (21,27).

Key words: optimal training loads, mathematical model, optimal control methods.

Introduction

The highest feats in sports are the realm of the few, whose born talents have found optimal conditions for development and improvement. A set of physical and psychosomatic features, programmed in genetic material of the individual, is a gift of the Nature. The degree of manifestation of these features depends on a set of external factors, which help or hinder the full development

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of constitutional features (Milicerowa 1978). Advantageous living condition may cause development of talents, however even small transgresses in this process lower possibilities of development of talented athlete, and may even totally stop it (Starosta, Handelsman 1990). Specially important for the development of separate talent and abilities of the athlete is proper dosage of training load. It is a basis of guiding the process of competitive preparation at each stage of training, deciding on the effectiveness of training.

One of greatest problems encountered in the course of planning training process is understanding mechanisms of adaptive processes to high training loads. The major issue is here the number of variables influencing given parameter (level of strength, speed, oxygen ceiling, etc.). Another important problem is time-limited effect of given stimulus, connected with process of effort adaptation. In order to solve the first problem, most often deterministic model is formed, containing limited number of independent variables, with the aid of which behaviour of one or more investigated dependent variables (sport result) is determined, or their behaviour is predicted (Mader 1988). It is rare to find criteria containing influence of training stimulus and changes of adaptive processes as a function of time (Perl 1997). One possible solution of the above problem is creation of non-commonplace meta models in connection with an analysis of single cases and time-series investigation. Meta models are abstract mathematic models generally constructed with the use of neural networks. They contain only basic logic concepts, enabling explanation of the question of structural similarities of investigated mechanisms (biological systems, training systems) (Mester, Perl 1999).

Another approach to solution of training load structure is represented by Ryguła and Wyderka (1990 and 1993) and Ryguła and Wojtylak (1998), who use mathematical model in form of a set of differential equations. These authors believe that each equation should describe the dynamics of development changes of given person as a function of values of all analysed features (state variables), used controls (training loads) and of time t (duration of training: $t_2 - t_1$).

Taking into account approaching the limits of adaptation abilities of human organism (Dobrzański 1989, Milicerowa 1978), the fundamental problem of today sports seem to be finding a tool, enabling extra-verbal extra-intuitional) implementation of controlling sports training.

A question arises: are there possibilities of precise determination of the kind and approximate value of the indices, describing limits of mean value of loads rational for given age and sex, ensuring competition success?

In spite of many attempts of solution, no algorithm has yet been developed which would provide basis for precise determination of the value of training loads used, therefore the aim of this work is development of such mathematical model, which would enable to compute the optimal controls of sports training. The implementation of this task is connected with obtaining response to the following research problems:

1. Are there mathematical tools, that will show planning paths for optimal training loads of endurance formation?
2. In what way to determine parameters of training loads?

Research material and methods

Characteristics of investigated persons

In order to obtain responses to the above questions, pedagogic experiment was made. In the proper experiment were taking part 45 schoolchildren of seventh grade of elementary school in comparable biological age (Tanner 1963) and similar level of general efficiency and swimming skill. The experiment was conducted in the period of 9 months. It was preceded by initial two-year efficiency preparation. To the initial stage persons were qualified (60 boys) that previously did not participate in systematic sport activities and had low level of swimming skill. They participated in training sessions twice a week on swimming pool (45 minutes). In the second year additionally one a week in gymnasium for 90 minutes. Both in the first and second year the activities had general and complementary character, were characterised by great variety of means and methods used. The aim of swimming pool activities was to teach swimming with three basic styles: crawl, back stroke crawl and classic, with special emphasis on free style.

After the initial stage the individuals were jettisoned that did not show expected progress, were not interested in swimming training and who have shown high absence in training.

The course of experiment proper

The investigated persons took part three times a week in activities on swimming pool (45 minutes) and additionally two times a week in activities in gymnasium (1.5 h). The value of training load was changed in monthly cycles, both quantitatively and qualitatively. The training means used were recorded quantitatively and qualitatively. The load capacity was expressed by the time of exercise duration, while for the qualitative aspect a qualification has been adopted after Sozański, using the concept "intensity range" (Sozański 1989). The measure of total load was the sum of work times with the use of specified training means with specified intensity range. The loads were chosen randomly according to the assumed limitations.

For rationalization of recording forms and at the same time exercise identification, a simplified catalogue of used training means has been created, which in further part of the work played a role of decision variables. It enables to use numerical symbols during data acquisition.

Means used on land:

1. General development exercises: elements of gymnastics, athletics, typical strength exercises (intensity range 4) - U_1.
2. Continuous runs, running plays, team plays, movement plays (intensity range 3) - U_2.
3. Means used in water:
4. Speed and reaction time exercises: distances up to 25 m, starts, turns, diving (full break) (intensity range 5) - U_3.
5. Exercises in anaerobic lactic acid range: distances 25 - 50 m, swimming with the use of arms or legs only and in coordination of these elements. Full rest periods were used where possible (intensity range 4) - U_4.
6. Exercises in mixed range: distances 100 - 200 m, swimming with arms, legs and in coordination, intermediate break (intensity range 3) - U_5.
7. Exercises in oxygen range: distances 400 - 800 m, short break (intensity range 2) - U_6.
8. Swimming-in, free swimming (intensity range 1) - U_1.
9. Exercises improving swimming technique (intensity range 2) - U_8.
10. Exercises teaching techniques (intensity range 1) - U_9.
11. Competitions - U_{10}.

Observed and measured variables

During the experiment the following characteristics (variables) were measured, playing the role of state variables: swimming on distance 800 m - X_1, swimming on distance 25 m - X_2, body height [cm] - X_3, body mass - X_4, length of lower extremities [cm] - X_5, length of upper extremities [cm] - X_6, oxygen efficiency determined on the basis of VO_2max [$ml \cdot kg^{-1} \cdot min^{-1}$] - X_7, anaerobic efficiency determined by maximum power obtained in Wingate test [W/kg] - X_8, breathing capacity [cm^3] - X_9, explosive strength - static broad jump [cm] - X_{10}, relative strength - measured by test of bar hanging ("Eurofit") - X_{11}, speed - measured as time of 10 m run - X_{12}, swimming step [number of full strokes on 25 m distance] - X_{13}.

The state variables were measured in semiannual periods, during the initial training and before the experiment proper and after each monthly training period.

Quality index

To build a model, it is necessary to choose from the measured variables one, called the quality index, the value of which at the end of training cycle should be maximal (minimal). In further discussion it was assumed, that it will the result in 800 m swimming, calculated in points according to table of Polish Swimming Federation. The measurements were made with attention to the starting requirements.

Investigation results

Chosen parameters of descriptive statistics of investigated group are shown in Table 1.

Table 1. Statistical characteristics of state variables of investigated athletes before the beginning of the experiment.

State variables	$\bar{x}$	S	A_s	Ku	V
Body height [cm]	164,2	7,37	1,456	2,824	4,48
Body mass [kg]	50,73	7,76	-1,081	2,591	15,20
Length of upper extremities [cm]	73,49	3,94	0,952	2,171	5,36
Length of lower extremities [cm]	79,15	4,19	1,259	2,651	5,29
Vo_{2max} [L/min]	3,15	0,52	-1,721	2,981	16,50
Maximum power [W/kg]	9,21	1,49	1,081	2,191	16,20
Breathing capacity [cm^3]	3027,5	243,59	2,001	1,852	8,04
Static broad jump [cm]	199,15	24,46	1,061	2,856	12,30
10 m run [s]	1,50	0,21	-1,536	2,896	42,00
Lifting on a bar [number of repetitions]	4,40	1,31	1,826	1,009	29,80
Swimming step [number of full strokes/25 m.]	14,66	2,66	1,081	2,007	18,10

$\bar{x}$ - mean, S - standard deviation, A_s - asymmetry index,
Ku - kurtosis index, V - variability index.

Construction of mathematical model

According to the methodology of action during solution of optimization tasks, at first it form as a set of differential equations has been justified. The value of each state variable (physical condition, sports results) at the end of certain unit of time is a function of general condition of the athlete, his results, described by state variables at the beginning of this unit of time and assumed training, that is implemented intensity of separate training means (controlling variables) in unit of time being analyzed. The aim of optimization of endurance formation is to determine such sequence of controls $U^{(0)}, \ldots , U^{(N-1)}$, to obtain the extremum of certain state variable (endurance) at the end of total training period. To enable the use of optimization methods and automate construction of the model, the Φ function was described as Taylor function expansion. To enable using methods of determination of optimal control according to Pontriagin principle, the model was made continuous and increments in training periods were replaced with derivatives.

For further discussion we have assumed to have at our disposal measurements of n state variables and m training means. Henceforth we have adapted the denotations for state variables vector $X(t)=(x_1(t),...,x_n(t))^T$, for controlling variables vector $U(t)=(u_1(t),...,u_m(t))^T$. The upper index ^{T} denotes transposition of vector, matrix. According to formerly developed method of investigation of sport training models (Ryguła, Wyderka 1993), as a mathematical model of endurance formation, a set of differential equations has been adapted in the form:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij} x_j + \sum_{j=1}^{n}\sum_{k=1}^{m} b^i_{jk} x_j u_k + \sum_{j=1}^{m} c_{ij} u_j + d_i + h_i(t,U) \qquad i=1,...,n \qquad (1)$$

where: x_i is the i-th state variable, i=1,...,n,
u_j is the use of j-th training means, j=1, …, m,
uj ` [0,1], 0 denotes lack, 1 - maximum use of j-th training means.

This equation was analyzed in range [0, T], where 0 is conventional beginning, and T - end of training period. It was therefore assumed, that optimization problem described in this work has the following form: Determine time functions of controlling variables $U(t)=(u_1(t),...,u_m(t))^T$,$u_i(t) \in [0,1]$ i=1,...m, $t \in [0,T]$ in such way to maximize $x_1(T)$, where $X(t)=(x_1(t),...,x_n(t))^T$ is the solution of (1) for the initial condition $X(0)=X_0$. X_0 is initial state of the athlete for whom we attempt to determine optimal training. The choice of the first state variable for maximization is dictated by simplification of further transformations. This problem is known as Mayer's problem [30]. $Z=[0, 1]^m$ set (Cartesian product) will be a set of permissible solutions. It is a compact and convex set. To solve the set, approximate methods were used. In part the Krylov-Czernousko algorithm was used to find optimal control (Rygula, Wyderka 1993).

Determination of model parameters

To construct a model, *n* state variables were chosen and *m* controlling variables and *z* athletes, for whom these parameters were measured or determined in period $[t_1, t_2]$. Further, the time period was transformed into $[0, T]$, where $T=t_2 - t_1$. τ denotes basic unit of time and $T=N\tau$. It was assumed that each athlete has all measurements in each time unit. The existing lack of data was completed by interpolation with glued functions. The measurements were denoted:

$\xi_i^l(s)$ - value of i-th state variable for l-th athlete in s-th time unit,

$\tilde{\theta}_j^l(s)$ - value of j-th state variable for l-th athlete in s-th time unit,

i=1,...,n, j=1,...,m, l=1,...,z, s=0,...N.

For further computations $\tilde{\theta}_j^l$ - defined as time of doing certain group of exercises during training - has been transformed into [0, 1] interval. In order to formulate differential model for computation optimal control, $a_{ij}, b_{jk}^i, c_{ij}, d_i$ numbers and function h_i were determined for i,j=1,...,.n, k=1,..,m. For this purpose a set of linear equations was formulated:

$$\xi_i^l(s+1) - \xi_i^l(s) = \sum_{j=1}^{n} a_{ij}\xi_j^l(s) + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^i \xi_j^l(s)\theta_k^l(s) + \sum_{j=1}^{m} c_{ij}\theta_j^l(s) + d_i \qquad (2)$$

s=0,...,N-1, l=1,...,z,i=1,...,n

It is a set of N*z*n equations with n*(n+n*m+m+1) unknowns. For the formulation of the problem to make sense, the following inequality must be true: N*z>n+n*m+m+1.

In such is the case, we have overdetermined set (Gordon et al. 1973). This inequality is necessary, for the model must average results measured for separate athletes.

The solution of (2) was sought in mean square sense. In another words, for each i such set of variables $a_{ij}, b_{jk}^i, c_{ij}, d_i$ i,j=1,...,.n, k=1,..,m, to minimize quadratic form

$$H_i = \sum_{l=1}^{z}\sum_{s=0}^{N-1}((\xi_i^l(s+1)-\xi_i^l(s))-(\sum_{j=1}^{n}a_{ij}\xi_j^l(s)+ \quad i=1,...,n \quad (3)$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{m}b_{jk}^i\xi_j^l(s)\theta_k^l(s)+\sum_{j=1}^{m}c_{ij}\theta_j^l(s)+d_i))^2$$

Minimum of quadratic form was found by equating partial derivatives H_i to zero in respect of separate parameters. n sets were obtained of (n+n*m+m+1) equations with (n+n*m+m+1) unknowns. The solution of each of these sets was substituted into (3) and differences of both sides were computed.

In practice, most difficult computational problem was to determine the maximum of the Hamilton function. The main reason were existing dependences between training means in most of known cases. For instance, the number of training means is limited by duration of training, some training means are mutually exclusive during the same training session.

In the way described above, the mathematical model was constructed, including seven equations.

The quality of fit with measured data was determined by computing the value from equation:

$$\delta_i = \frac{H_i}{\sum_{l=1}^{z}\sum_{s=0}^{N-1}((\xi_i^l(s+1)-\xi_i^l(s))^2} \quad i=1,...,n \quad (4)$$

The δ_i value has been named coefficient of fitting. The calculated coefficients of fitting model to data are multidimensional equivalent of relative error (ratio of difference of accurate and approximate value to accurate value times 100). Results are shown in Table 2.

Table 2. Coefficients of fitting for separate equations.

Equation for variable	X1	X2	X3	X4	X5	X6	X7
δ_i [%]	5,5	4,0	6,8	5,5	4,9	1,2	4,4

The values of the coefficients of fitting are very small, indicating good accuracy of model. It was therefore assumed that model sufficiently approximates experimental data.

Table 3. Evaluation of the influence of the value of controlling variables on increase of state variables

State variables	Controlling variables									
	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}
X_1	-140.61	**72.98**	**53.52**	-44.56	23.69	18.42	-16.51	-44.71	-21.72	**65.15**
X_2	-584.41	-87.40	-8.17	82.09	-1.13	116.30	242.39	378.89	-54.81	8.50
X_3	14.07	11.25	-2.80	-17.74	-1.74	0.52	-2.52	-8.87	3.98	-2.86
X_4	-12.78	-2.02	-4.63	-2.63	-0.54	-0.91	5.36	14.10	-2.72	4.68
X_5	7.28	-5.98	-2.70	2.28	-0.50	-1.05	1.44	5.96	-2.00	-3.11
X_6	8.63	-2.81	3.32	4.93	1.78	3.79	0.97	-7.48	-2.79	-5.94
X_7	-604.73	137.47	722.07	294.65	171.21	486.91	45.79	-1004.7	25.41	-94.86

By analysis of Table 3, the increase of state variables may be evaluated in dependence from controlling variables. Great influence on the improvement of result in 800 m swimming has U_2, U_3 and U_{10} variables. U_2 variable is land exercises, the aim of which is to form oxygen energetic mechanisms. Their share (energetic processes) in covering energy demands during swimming on the said distance is about 80-90% (Costill at al.1995, Counsilman 1977, Paradowski 1991, Pietrusik 1981). For this reason, using these exercises will lead to improvement in 800 m swimming result. Variable U_3 denotes exercises aimed at formation of speed skills, having importance in swimming (Raczek 1989). U_{10} variable denotes participation in competitions. Method of control starts is recognized method of preparation to competition season, used by numerous athletes (Bulgakova 1996, Colwin 1992, Platonov 1997, Raczek 1989). It is a kind of effort most closely approaching the most important competition with its internal and external structure. For this reason, a great influence of this variable on the improvement of results seems to be well founded. Some controversy arises however from the negative influence of U_1 controls. General development exercises are the basis of sport training. Perhaps in the investigated group the possibilities of result improvement by the use of general development exercises are nearly exhausted and in further training microcycles the work should be directed on activities leading to further specialization.

Optimal control

The most evident gain from the use of model is determination of optimal control (training leading to best competition results) for given initial conditions for specified athlete. The athlete has been randomly chosen for analysis. Henceforth he will be referred to as athlete X_p.

In view of the fact, that for numerical solution of model equation a step equal to half of observation period has been chosen, optimal control was determined with interval of two weeks. The values of separate controlling variables therefore determine total time of doing given exercise during two weeks.

Table 4. Controls leading to maximum result expressed in points on 800 m distance for athlete X_p.

Week	Training means										800 m
	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}	result
1-2	45	90	21	0	0	45	23	45	0	2	66.9
3-4	55	90	25	0	0	55	28	55	0	2	73.8
5-6	55	90	25	0	0	55	28	55	0	2	80.5
7-8	55	90	25	0	0	55	28	55	0	2	86.9
9-10	55	90	25	0	0	55	28	55	0	2	93.1
11-12	55	90	25	0	0	55	28	55	0	2	99.1
13-14	80	158	0	0	200	0	40	40	40	4	104.5
15-16	85	158	0	136	0	0	38	38	38	4	109.0
17-18	75	135	0	0	138	0	38	38	38	5	113.4
19-20	60	158	0	96	0	0	30	30	30	3	117.9
21-22	70	158	0	0	140	0	40	40	40	5	122.4
23-24	60	135	0	0	140	0	30	30	30	4	127.1
25-26	60	135	0	0	140	0	30	30	30	4	133.4
27-28	45	68	0	0	113	0	23	23	23	2	139.3

The results of athlete X_p with actually implemented training are shown in Table 5. It can bee seen that the actual training of the athlete was very close to optimal for maximizing the 800 m swimming result.

Table 5. Comparison of the actual results with results obtained from model at different optimizations for athlete X_p.

Max. of 800 m result	State variables						
	X_1	X_2	X_3	X_4	X_5	X_6	X_7
	139	315	196	13	143	32	2790
Actual result	**105**	**191**	**190**	**16.0**	**140.0**	**30.0**	**2500**

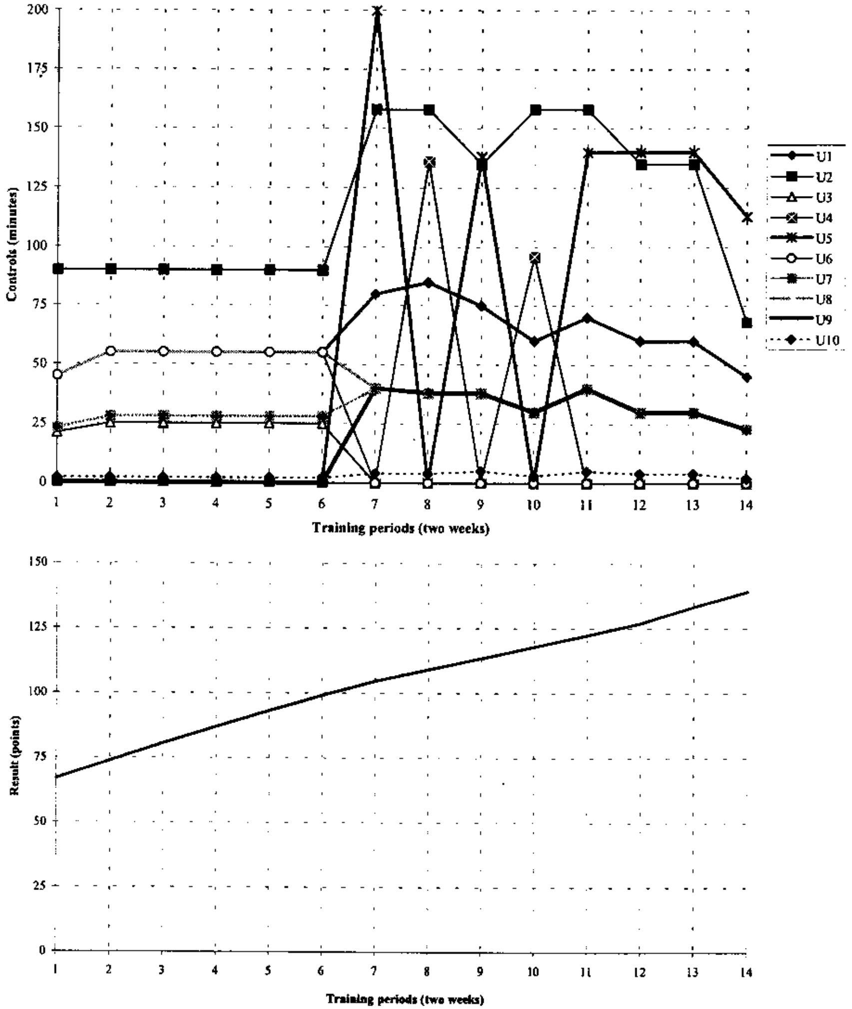

Figure 1. Optimal control of 10 training means (upper graph) and corresponding value of quality index (800 m result) for chosen athlete (lower graph).

The most important element of the investigation was determination of optimal values of controls in swimming training, directed to obtain maximal result in 800 m swimming. To obtain the expected result - optimal training - a model has been formulated, beginning from its shape (it was determined what is state variables, what is controls, which form have relationships of increments from state variable and control), ending with determination of numerical values of all its parameters.

The basis for determination of detailed model was pedagogical experiment made with the group of 14-year boys. Before starting the experiment it has been decided which variables will be observed. Such elements of training were determined as can be expressed in numbers and changed during its duration. During the training the training loads of separate athletes were changed in a way providing most information in observation material. The collected data has been processed.

The form of model suggested in the work is nonlinear, most simple of all. It is possible and advisable to use more refined methods of mathematical statistics.

The work demonstrates basic possibilities of using mathematical model in sport training:

- Evaluation of the influence of different controls on sport result;
- Determination of optimal training controls for different athletes.

The analysis of the optimal shapes of controlling functions has shown that they are variable in time. It should be noted that each training period is characterized with variable controls. The range of variability is very great; proportions of used controls also differ greatly in dependence of training period.

The results of the investigation, as well as experiences of other authors indicate that optimization implicates individualization of training of children and youngsters (Filin 1986, Komor 1984, Kozioł 1985, Ryguła 1996, Ważny 1978). For each individual exists a determined, individual volume and intensity of training, absolutely necessary to keep effort level of physiological characteristics on hitherto existing level. This applies to each stage of training where the workload should be compatible with given period of development and assumed sporting aim of particular individual (Filin 1986).

The results of investigation have confirmed theoretical consideration of Szczotka (1980), indicating possibility of controlling the training, when the improvement of result depends on the time of use of separate training means. Similar to the investigation of mathematic model of high jump, Ryguła and Wyderka (1996) have found great practical importance of the developed model*.

Comparing results of investigation on determination of optimal control with investigation of Gordon et al. (1973), it should be said that construction of mathematical model is a helpful tool serving non-intuitive control of training process. It enables to compute optimal strategy for athletes preparing for competition. The investigation of Gordon was applied to athletes of high class. The considerations presented in this work apply to athletes of small training experience, therefore it seems to be little point in comparing the computed controls. It is also difficult to discuss the application of meta-models as our knowledge of them is so far insufficient. It seems however that it is worthwhile to undertake investigations of verification of these models in the scope of training loads structure.

Conclusions

The measurement results collected in the experiment and subjected to statistical-mathematical analysis entitle to formulate the following conclusions:

- Basic mathematical tool, which may be used for planning training loads, is optimal control theory, on the basis of which a mathematical model was constructed and determined. Development of this tool has enabled the evaluation of the influence of chosen variables on the sport result;
- The mathematical model should be a set of differential equation. Each equation should describe the dynamics of development of given characteristic in dependence of all analyzed characteristics (state variables), used controls (training means) and ratio of state variables and controls;
- On the basis of computed mathematical model such course of changes of control parameters may be determined, that maximizes final effect of increasing sport result.

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* In the model developed by them only two controlling variables were used: percentage of intensity in the scope of mechanical work and percentage of intensity in the scope of mechanical power.

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