SPEED CAPABILITIES IN SWIMMING IN VIEW OF OPTIMAL CONTROL THEORY

by

IGOR RYGUŁA*

The author tries to help the attempts to solve certain optimization dilemma found not only in sport sciences, namely decision: which size of stimulus, quantitatively and qualitatively (e.g. training load) is best for the organism of the athlete. Pressure in the area of international sport competition is so big that in the choice between reasonably graded reduction of load with increase of regeneration effects on the one hand and further increasing of load according to rule "more helps more" on the other hand, it generally forces the choice of greater load. Fundamental here is a fear that the competition also exercises more and after possible defeat insufficient exercise will be pointed as a cause (Mester and Perl 2000). One way to solve such dilemmas is mathematical description of the analyzed phenomenon in form of a mathematical model. Possession of such tool enables to determine the best solutions in a sense of assumed criterion.

The paper points out that basic problem causing that sport sciences cannot satisfactorily solve this problem is the complex nature of the phenomenon of psychophysical fitness, variegation of separate parameters and their changing action. So far it was not possible to contain in a model all variables influencing the fitness or parameters decisive for fitness in a way enabling transferring them to training with insignificant errors. Representation of time function, adaptation hiding behind these parameters, is extremely difficult (Maas and Mester 1996).

Key words: physiological adaptation, mathematical model, optimal training loads.

Introduction

The value of each state variable (physical condition of the athlete, score) at the end of given time unit is a function of general condition of the athlete, his results, described with state variables at the beginning of this time unit and

* Assist. Prof., Dep. of Theory of Sports, Acad. of Phys. Educ., Katowice
Correspondence address: 72a Mikolowska Str. 40-065 Katowice, Poland

chosen exercise, that is the implemented intensity of separate training means (control variables) in analyzed time unit. In symbolic notation:

$$X^{(l+1)} = X^{(l)} + \Delta X^{(l)} \quad \text{gdzie} \quad \Delta X^{(l)} = \Phi(X^{(l)}, U^{(l)})$$

where l denotes number of training period, l = 0, ..., N-1.

The aim of optimizing swimming training is to find a sequence of controls $U^{(0)}, \ldots, U^{(N-1)}$, to obtain extreme of certain state variable (score, physical fitness) at the end of whole exercise period $X^{(N)}$=max (min).

Generally, determination of Φ function is impossible. Its approximation may be implemented for given group of athletes, for which the measurement results both of state variables and control variables are known. To be able to use optimization methods and automate the development of the model, Φ function should be approximated with possibly simple formula. We suggest the form: Φ(X,U)=aX+cU+bUX+d. It is a part of Taylor series expansion of Φ function. In the right-hand side of the above formula, the first component (aX) may be interpreted as description of the influence of athlete's condition on his or her score, second, (cU) - as description of the influence of exercise, third (bUX) - of the influence of joint condition and exercise.

To be able to use principles of finding optimal control according to Pontriagin principle, the model should be made continuous and the increments in exercise intervals replaced with derivatives.

Moreover, in the following we will be considering vectors of state variables and vectors of controls:

$$\hat{X},\ \hat{U}$$

Let's assume that we have at our disposal the measurements of *n* state variables and *m* training means. We will denote the state variables vector $X(t)=(x_1(t),...,x_n(t))^T$, and control variables vector $U(t)=(u_1(t),...,u_m(t))^T$. The upper index ^{T} denotes transposition of vector, matrix.

According to former considerations [15], as a model of sport training a set of differential equations has been chosen in the form:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^{i} x_j u_k + \sum_{j=1}^{m} c_{ij}u_j + d_i + h_i(t,U) \quad i=1,...,n \qquad (1)$$

where: x_i is i-th state variable, i=1,...,n,

u_j is the use of j-th training means, j=1,...,m.,

$u_j \in [0,1]$, 0 - none, 1 - maximal possible utilization of the j-th training means.

This equation will be analyzed in the interval [0,T], where 0 is assumed beginning, and T is the end of exercise period. It is also possible to place in a model the absolute values of training means such as duration of certain exercise in training. In this case it is necessary to define lower und upper bound for such control. The problem of optimization we are interested in has the form:

Determine time functions of control variables $U(t)=(u_1(t),...,u_m(t))^T$, $u_i(t) \in [0,1]$ i=1,...m, $t \in [0,T]$ in such way to maximize $x_1(T)$, where $X(t)=(x_1(t),...,x_n(t))^T$ is a solution of (1) for the initial condition $X(0)=X_0$. X_0 is the initial state of the athlete, for whom we aim to determine optimal exercise. The choice of the first state variable for maximization is dictated by simplification of further calculations. This problem is known as Mayer problem (Legras 1974, Leitmann 1972). A set $Z=[0,1]^{m.}$ (Cartesian product) will be a set of permissible controls. It is compact and convex set. The limitation of the problem to determination of the maximum is simple to analyze. In case when we want to minimalize a value of certain state variable (e.g. score in 100 m swimming), this value is placed with the opposite sign, or the initial condition in conjugated system is changed from 1 to –1.

In following analysis we will use expression $F=F(t,U,X)=(F_1(t,U,X),...,F_n(t,U,X))^T$, where F_i denotes right-hand side of the i-th equation of the set (1),

$$F_i(t,X,U) = \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^{i} x_j u_k + \sum_{j=1}^{m} c_{ij}u_j + d_i + h_i(t,U) \quad i=1,...,n$$

In view of the form of the right-hand side of the set (1), it is very difficult to use analytical solution (function e^A). For this reason to solve (1) approximate methods are often used (e.g. Euler method).

1. Solution of mathematical model

The problem of training optimization with an accent on the development of the speed of young swimmers has been formulated in the form known in literature (Mayer problem). Because of this we may positively decide a problem of existence and uniqueness of the optimal control (Legras 1974). Because its analytical form is very difficult to determine, we will use approximate methods.

According to (Ryguła and Wyderka 1990), Hamilton function needed for determination of optimal control has the form:

$$H(t,X,U,\Psi)=\langle F(t,X,U),\Psi(t)\rangle \qquad (2)$$

where symbol $\langle u,v \rangle$ denotes scalar product of vectors $\langle u,v\rangle=\sum_{i=1}^{n} u_i v_i$

Function Ψ present in (2) is solution of so-called conjugate system:

$$\Psi'=W\Psi,\ \Psi(T)=(1,0,...,0)^T \qquad (3)$$

where W is a matrix with components

$$w_{ij}=-\frac{\partial F_j}{\partial x_i} \quad i, j=1,..,n.$$

Effecting necessary transformations we obtain

$$w_{ij}=-a_{ji}-\sum_{j=1}^{m} b_{ik}^{j} u_k \qquad i, j=1,...,n$$

As in case of set (1), to solve (3) we will use numerical methods. According to Pontriagin maximum principle [16], set (1) and (2) and Hamilton function (2) suffice to determine optimal control. A method most suitable for described model is Krylov-Czernousko algorithm (Legras 1974, Ryguła and Wyderka 1993).

2. Determination of model parameters

In stage I we have chosen *n* state variables and *m* control variables and *z* athletes, for who these parameters were measured or determined for time period $[t_1, t_2]$. Next we transform time interval to [0, T], where $T = t_2 - t_1$. Let τ denote basic unit of time and $T = N\tau$. We assume that each athlete has all measuments in each unit of time. When data are incomplete, they may completed with interpolation. Let's denote measurements

$\xi_i^l(s)$ - value of the i-th state variable for the l-th athlete in the s-th unit of time,

$\tilde{\theta}_j^l(s)$ - value of the j-th control variable for the l-th athlete in the s-th unit of time,

i=1,...,n, j=1,...,m, l=1,...,z, s=0,...N.

In case when $\tilde{\theta}_j^l$ is defined as duration of certain group of exercises during the training, these data must be transformed into [0,1] range. We may do this in the following way:

for each variable, that we consider to be control variable we define its interval $[\delta_j,\gamma_j]$ j=1,...,m,

we make the transformation $\theta_j^l = \dfrac{\tilde{\theta}_j^l - \delta_j}{\gamma_j - \delta_j}$ j=1,...,m, l=1,...,z.

In order to formulate a differential model for computing optimal control, numbers $a_{ij}, b_{jk}^i, c_{ij}, d_i$ should be determined as well as function h_i for i, j=1,...,.n, k = 1,.., m. For this purpose we form a set of linear equations

$$\xi_i^l(s+1) - \xi_i^l(s) = \sum_{j=1}^{n} a_{ij}\xi_j^l(s) + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^i \xi_j^l(s)\theta_k^l(s) + \sum_{j=1}^{m} c_{ij}\theta_j^l(s) + d_i \qquad (4)$$

s = 0,...,N-1, l=1,...,z, i=1,...,n

This is a set of N*z*n equations with n*(n+n*m+m+1) unknowns. To make the problem sensible there must be the inequality N*z>n+n*m+m+1.

In this case we are dealing with so-called overdetermined set (Perl 1997). Satisfying this inequality is necessary, because the model must average results measured in separate athletes. The set (4) may be decomposed into *n* set, one for each state variable. The matrices of each set will be the same. The right-hand sides of these sets will be different. We will be seeking solution of (4) in root mean square sense. In other words, for each i we are seeking such set of variables $a_{ij}, b_{jk}^i, c_{ij}, d_i$ i, j = 1, ..., n, k = 1, ..., m, that minimize quadratic form

$$H_i = \sum_{l=1}^{z}\sum_{s=0}^{N-1}((\xi_i^l(s+1) - \xi_i^l(s)) - (\sum_{j=1}^{n} a_{ij}\xi_j^l(s) + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^i\xi_j^l(s)\theta_k^l(s) + \sum_{j=1}^{m} c_{ij}\theta_j^l(s) + d_i))^2 \qquad (5)$$

i = 1,..., n

Minimum of quadratic form is found be equating partial derivatives H_i to zero for separate parameters. We will then obtain *n* sets (n+n*m+m+1) of equations with (n+n*m+m+1) unknowns. Solution of each of these sets should be then entered into (4) and compute differences of left-hand and right-hand side. If the difference is rather big, we may try to approximate this difference with function $h_i(t,U)$. Its form may however be determined for given cases by observation of time runs of differences in set (4).

Correct construction of separate sets of equations is very important. Let's determine, that $1 \le i \le n$. Let's denote $\Delta\xi_i^l(s) = \xi_i^l(s+1) - \xi_i^l(s)$ l = 1,..., z, s = 0, ..., N − 1. The first *n* equations resulting from the relation $\frac{\partial H_i}{\partial a_{ip}} = 0$, p = 1, ..., n will have the form:

$$\sum_{j=1}^{n} a_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \xi_p^l(s) + \sum_{j=1}^{n} \sum_{k=1}^{m} b_{jk}^i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \xi_p^l(s) * \theta_k^l(s) +$$
$$\sum_{j=1}^{m} c_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \theta_j^l(s) * \xi_p^l(s) + d_i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_p^l(s) \quad = \quad \sum_{l=1}^{z} \sum_{s=0}^{N-1} \Delta\xi_i^l(s) * \xi_p^l(s)$$

Next n*m equations resulting from relation $\frac{\partial H_i}{\partial b_{pq}^i} = 0$ p = 1, ..., n, q = 1, ..., m, have form:

$$\sum_{j=1}^{n} a_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \xi_p^l(s) * \theta_q^l(s) + \sum_{j=1}^{n} \sum_{k=1}^{m} b_{jk}^i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \xi_p^l(s) * \theta_k^l(s) * \theta_q^l(s) +$$
$$\sum_{j=1}^{m} c_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \theta_j^l(s) * \xi_p^l(s) * \theta_q^l(s) + d_i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_p^l(s) * \theta_q^l(s) =$$
$$\sum_{l=1}^{z} \sum_{s=0}^{N-1} \Delta\xi_i^l(s) * \xi_p^l(s) * \theta_q^l(s)$$

Next m equations resulting from $\frac{\partial H_i}{\partial c_{iq}} = 0$ q = 1, ..., m have form:

$$\sum_{j=1}^{n} a_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \theta_q^l(s) + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \theta_k^l(s) * \theta_q^l(s) +$$
$$\sum_{j=1}^{n} c_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \theta_j^l(s) * \theta_q^l(s) + d_i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \theta_q^l(s) =$$
$$\sum_{l=1}^{z} \sum_{s=0}^{N-1} \Delta\xi_i^l(s) * \theta_q^l(s)$$

The last equation resulting from relation $\frac{\partial \mathrm{H_i}}{\partial \mathrm{d_i}} = 0$ has the form:

$$\sum_{j=1}^{n} a_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) + \sum_{j=1}^{n}\sum_{k=1}^{m} b_{jk}^i \sum_{l=1}^{z} \sum_{s=0}^{N-1} \xi_j^l(s) * \theta_k^l(s) + \sum_{j=1}^{n} c_{ij} \sum_{l=1}^{z} \sum_{s=0}^{N-1} \theta_j^l(s) + d_i = \sum_{l=1}^{z} \sum_{s=0}^{N-1} \Delta\xi_i^l(s)$$

Using the fact that matrices of the equations are the same for all state variables, we can solve these sets simultaneously, using Gauss-Jordan elimination (Legras 1974). Practically, the greatest computational difficulties are encountered in determination of the maximum of Hamilton function. The main cause is the existence of relations between training means, present in overwhelming majority of known cases. For instance:

- the sum of training means is limited by the duration of training;
- certain means are mutually exclusive during one training session.

The quality of model fitting to measured data may be computed with the formula:

$$\delta_i = \frac{\mathrm{H_i}}{\sum_{l=1}^{z} \sum_{s=0}^{N-1} ((\xi_i^l(s+1) - \xi_i^l(s))^2} \qquad i=1,...,n \qquad (6)$$

The quantity δ_i will be called fitting coefficient.

Material for investigation constituted 40 boys aged 14. They were subjected to two years of investigation, according to assumptions of the experimental model. The used scheme of investigation was $RX_n^n Y_n$, that is one dependent variable (Y_n), n independent variables (X_n) using randomization principle (R). The results of two-year observation[*] constituted the basis of contruction of the mathematical model of the development of speed of exercising swimmers.

During the experiment the following characteristics (variables) were measured:

1. State variables:
   – height [cm],
   – body mass [kg],
   – length of lower limbs [cm],
   – length of upper limbs [cm],
   – oxygen efficienty evaluated on the basis of Vo_2max [l/kg * min],
   – anaerobic efficienty evaluated by maximal power obtained in Wingate test [W/kg],
   – vital capacity [cm^3],
   – static wide jump [cm],
   – time of 10 m run [s],
   – hoists on bar [N cycles],
   – swimming step [number of full cycles on the 25 m distance].
2. Decision variables: total duration of exercise in separate training cycles, 10 training means.

To construct a model, it is necessary to chose from the measured variables one variable, called quality index, the value this variable should be maximal (minimal) at the end of training cycle. For the purposes of further analysis, the score in 25 m free style swimming has been chosen. The time of swimming this distance was expressed in points using the tables of Polish Swimming Association. Because the tables contain no data for 25 m distance, the actual scores were multiplied by 2 and table for 50 free style swimming was used.

As control variables, the following training means were assumed in the model:

U_1 - general development exercises, [min],

U_2 - forms of runs, jumps and field games [min],

---

[*] More detailed description of the experiment and specified variables is contained in (Cholewa 1998)

U_3 - swimming distances up to 25 m, full break [min],
U_4 - swimming distances 25-50 m, full break [min],
U_5 - swimming distances 100-200 m, incomplete break [min],
U_6 - swimming distances 400-800 m, short break [min],
U_7 - general swimming, compensation [min],
U_8 - exercising style techniques [min],
U_9 - exercising and teaching of jumps and reversals [min],
U10 - competitions [min].

Model construction

We have decided to use the first seven state variables and all controls for the construction of model. A smaller number of variables facilitates interpretation of the results. Making model more compact helped to shorten the calculations and making more optimizations.

In accordance with the above procedure, the values of all coefficients in equation (1) were firsts calculated. Next, a mathematical model was constructed as a set of seven differential equations with 10 control parameters. They presentation has been omitted from this work.

Results of the investigation

Comparison of different training variants

A model should serve mainly the individualization of training process, i.e. it should be used for comparison of different ways of solving training process for the same athlete. This is illustrated by the following example:

- One athlete has been chosen and his initial conditions were entered into the model,
- Different courses of exercise were defined, for instance all that were used during data collection for different athletes,
- For each training the results were calculated after 24 months.

Table 1 presents the results of such procedure. The athlete was chosen randomly for analysis. He will be further called athlete Z_d. Before the start of the experiment, he had results better then the group average.

Results, computed for athlete Z_d with his actual training are in the first row of the table. As can be seen from the table, depending on chosen training we encounter high spread of scores. This analysis enables choosing the way of training, used before, bringing most advantages. An analysis of results in Table 1 has shown that training used for athlete Z_d has greatly differed from optimal for this athlete for maximizing score of 25 m swimming. Our chosen athlete would obtain the best score if he were trained with training variant no. 27.

Table 1. Results for athlete Z_d after 24 months of training with different training variants.

Training number	State variable						
	X1	X2	X3	X4	X5	X6	X7
1	2	3	4	5	6	7	8
1	208.2	105.0	195.5	15.6	141.2	30.5	2434.9
2	198.0	95.9	195.3	15.7	141.5	30.2	2398.0
3	226.7	104.9	195.6	14.4	140.9	30.9	2562.0
4	232.5	104.9	195.8	14.4	141.4	30.7	2519.0
5	118.3	90.5	191.0	18.0	139.5	31.2	2609.8
6	238.1	107.9	192.9	14.0	141.4	31.0	2564.7
7	184.9	90.5	193.6	15.5	141.0	30.9	2504.8
8	260.1	107.5	195.7	14.4	141.6	30.7	2457.3
9	154.4	82.8	195.6	16.6	140.2	30.4	2473.2
10	249.6	108.2	195.0	14.1	141.4	30.3	2504.6
11	129.2	80.4	195.1	17.4	139.5	29.5	2524.2
12	233.6	105.5	195.3	14.5	140.8	28.6	2494.0
13	121.8	71.5	189.5	16.8	140.3	30.0	2520.0
14	110.3	67.7	183.6	17.0	139.2	29.9	2620.7
15	144.9	72.3	194.3	17.2	140.1	30.3	2371.7
16	151.6	85.1	182.7	16.2	140.8	29.7	2532.4
17	151.5	82.1	190.5	16.5	140.8	30.2	2444.6
18	210.8	102.3	185.0	14.3	142.0	29.8	2573.6
19	100.6	64.8	187.0	17.8	140.3	30.1	2545.5
20	108.7	75.1	193.2	17.7	139.4	30.3	2519.3
21	135.2	70.4	190.8	17.0	140.8	29.2	2414.1
22	77.1	59.1	185.5	17.7	140.5	29.9	2624.5
23	227.7	92.0	193.7	14.3	141.9	29.1	2307.0
24	236.9	95.2	191.3	13.7	142.1	29.0	2405.2
25	136.3	73.5	184.9	16.0	140.5	29.9	2543.3
26	244.2	86.6	193.0	13.1	143.0	28.7	2357.3
27	**298.2**	94.8	194.5	13.1	143.0	27.4	2241.8
28	137.1	71.2	192.3	17.1	141.3	30.3	2514.1

Table 1

29	230.3	100.8	192.9	14.5	142.3	29.9	2359.5
30	181.9	87.2	195.0	16.4	141.5	30.5	2407.8
31	150.1	76.6	186.2	16.2	140.9	31.1	2385.0
32	162.3	79.1	188.1	16.0	141.4	31.1	2405.2
33	96.8	52.4	171.3	17.4	140.6	32.5	2473.9
34	121.5	63.7	169.3	15.8	140.9	31.8	2416.8
35	168.0	79.5	185.7	15.9	141.6	31.6	2419.8
36	124.2	62.5	179.3	16.4	140.9	32.0	2434.6
37	113.5	69.1	171.5	16.7	140.1	31.3	2543.9
38	110.1	65.8	178.0	16.5	140.5	31.3	2485.5
39	119.4	63.4	175.8	16.5	140.7	31.9	2443.9
40	126.9	66.7	176.8	16.5	140.7	32.0	2411.7
Actual result	**191**	**105**	**190**	**16.0**	**140.0**	**30.0**	**2500**

Optimal control

The most evident advantage of using a model is determination of optimal training (training leading to best scores) for defined initial conditions of chosen athlete. At first, the controls were computed for the athlete described in last section. The optimal control has been determined for maximizing point score on 25 m distance. Table 2 presents results theoretically possible in case of optimal training shown in Fig. 1.

Table 2. Comparison of actual scores and scores obtained froim model with different optimizations for athlete Z_d.

Maximum of 25 score	State variable						
	X1	X2	X3	X4	X5	X6	X7
Theoretical	368	121	196	14	143	30	2506
Actual.	**191**	**105**	**190**	**16.0**	**140.0**	**30.0**	**2500**

The optimal control was determined with the interval of 2 months. The values of the values of separate control variables therefore present total time of given exercise during two months.

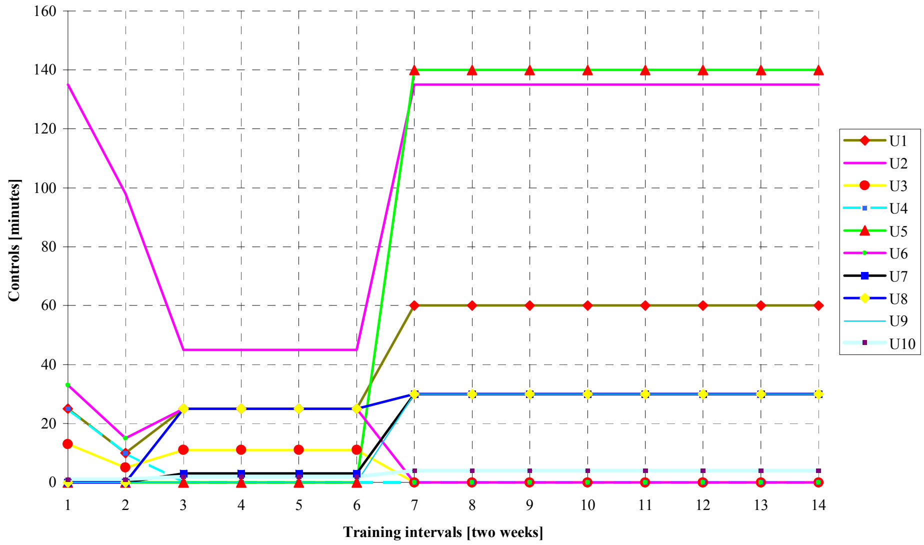

Fig. 1. Optimal controls of 10 training means (above) and respective value of quality index for chosen athlete (below).

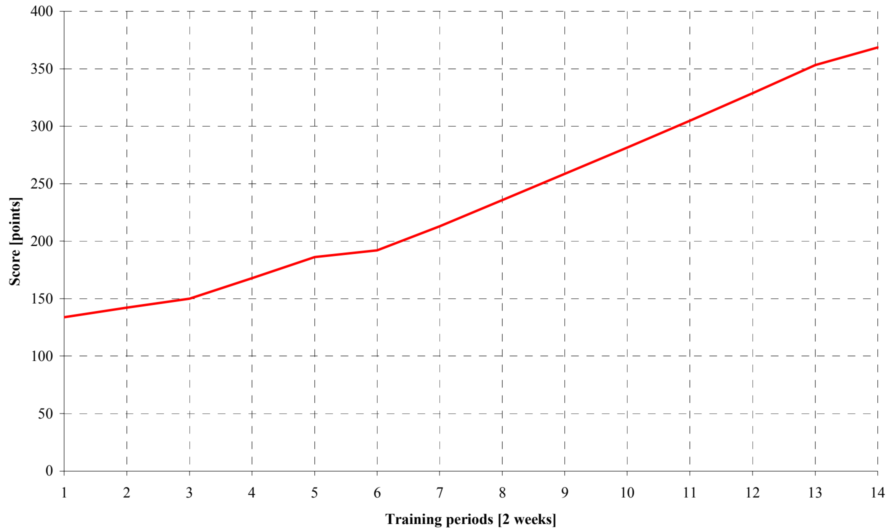

Table 3. Control leading to maximal point score on 25 m distance for athlete Z_d.

Months	Training means										25 m score
	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	
1-2	25	135	13	25	0	33	0	0	0	1	133.9
3-4	10	98	5	10	0	15	0	0	0	1	142.2
5-6	25	45	11	0	0	25	3	25	0	2	149.9
7-8	25	45	11	0	0	25	3	25	0	2	167.8
9-10	25	45	11	0	0	25	3	25	0	2	186.0
11-12	25	45	11	0	0	25	3	25	0	2	191.8
13-14	60	135	0	0	140	0	30	30	30	4	212.9
15-16	60	135	0	0	140	0	30	30	30	4	235.8
17-18	60	135	0	0	140	0	30	30	30	4	258.5
19-20	60	135	0	0	140	0	30	30	30	4	281.3
21-22	60	135	0	0	140	0	30	30	30	4	304.6
23-24	60	135	0	0	140	0	30	30	30	4	328.5

Discussion of the results

The most important element of the investigation made was determination of optimal controls for development of speed abilities of 14-year old swimmers, directed towards obtaining maximal result on 25 m distance. To obtain the required result -optimal training - a model was formulated, beginning with its shape (defining what constitutes state variables and what controls, which form have relations of increments from state variables and control variables) to determination of numerical values of all parameters of this model. The form of the model must be resultant of different opposing tendencies. The model must be complex, for all relations pertaining to human body are complex (Mader 1988, Mester and Perl 1999). At the same time, the model must be simple enough to enable using known algorithms. As the results of the research have shown, solution is possible, when certain limitations are used (set of limitations (Komor 1982, Mester and Perl 2000)).

The form of model proposed in this paper is nonlinear, most simple of all. At the same time, a simple form of conjugate set has been obtained. The paper shows simple, but very effective method of model construction on the basis of available data.

The uses of constructed model may be manifold. This work shows three of them that seem to be basic:

- Evaluation of the influence of different factors on the development and formation of speed capabilities (25 m score),
- Evaluation of the influence of defined training cycles for given athlete,
- Determination of optimal training controls for chosen athlete.

The analysis of the influence of state variables on the score has shown that great importance for obtaining high results in speed effort such as maximum speed swimming on 25 m distance, has the level of dynamic strength and somatic parameters. This is confirmed by many authors (Cholewa 1998, Counsilman 1977). Only parallel increasing of the level of strength and development of speed may lead to adequate raising of speed abilities in the framework of movement structure of given discipline. At the same time, body mass and height have great prognostic value for determination of speed capabilities of the athletes.

On the basis of evaluation of the influence of defined training cycles on the increments of analyzed characteristics of chosen athlete it was found out, that the spread of result is enormous. The effect of training is influenced not only by load used fir given exercise session, but also load used in former sessions (additive training effect). Therefore the use of various solutions in dosing of the effort brings different final effects (principle of equal final results). The use of model has enabled comparison of separate training periods from the point of view of their effectiveness and to chose the period that brings greatest advantages according to assumed criterion for given athlete.

The results of the investigation, as well as the experience of other workers indicate that optimization implies very desirable individualization of training of children and teenagers (Filin 1986, Ryguła 1999). For each individual exists defined, individual volume and intensity of training, absolutely necessary for keeping the effort level of physiological characteristics on the proper level (Nowikow et al. 1976). This applies to each stage of training, where work done should be adequate to given development period and assumed sport aim of given individual (Morawski 1980). Therefore it is necessary to determine optimal controls for given athlete, in our case we maximize point score on 25 m distance.

An analysis of optimal shape of control functions has shown that they are time variable. It is worth noting that each training period is characterized by changing controls. The range of changes is very great and proportions of used controls are different depending on training period.

Conclusions

1. Solution of the problem of optimization of training loads requires individual approach (mathematical description of this phenomenon and its solution), depending on many factors (sport discipline, level of biological development of individuals, their reactivity to training stimuli, etc.).

2. A great help in solving the problem of training loads may be the method of construction and solving mathematical models of elements of sport training, developed and sanctioned by this author.

3. The mathematical model, even the best, may constitute just a help for the coach. It should be however remembered that we may speak of controlling the training only when the training loads are computed with the use of mathematical model.

REFERENCES

Cholewa J.1998. *Mathematical model and optimal control of the training of 14-year old swimmers*. Katowice, AWF. Doctor Thesis (In Polish).

Counsilman J.1977. *Swimming Power*. Swimming World nr 10.

Filin V.P. 1986.*Vospitanie fiziceskich kacestv u junych sportsmenov*. Moskwa.

Komor J.A. 1982. *Application of modelling methods in sports*.Instytut Sportu, Warszawa (In Polish).

Legras I. 1974. *Practical methods of numerical analysis*. Warszawa, WNT (In Polish).

Leitmann G. 1972. *Introduction to the optimal control theory*. Warszawa, PWN (In Polish).

Maas S., Mester J. 1996. *Diagnosis of individual physiological response in elite sport by means of time-series-analyses*. (In:) Marconnet, P. et al. (Eds.): *Frontiers in Sport Science. The European Perspective*. University of Nice. Nice ,98-99.

Mader, A. 1988. A *Transcription-Translation activation feedback circuit as a function of protein degradation, with the quality of protein mass adaptation related to the average functional load.* Jour. Theo. Biol.134 , 2 – 17.

Mester J., Perl J. 1999. *Unconventional simulation and empirical evaluation of biological response to complex high training loads*. (In:) Parisi, P., Pigozzi, F., Prinzi, G. (Eds.): *Sport Science ´99 in Europe*. Rome, 163.

Mester J., Perl J. 2000. *Zeitreihenanalysen und Informatisches Metamodell zur Untersuchung physiologischer Adaptationsprozesse*. Leistungssport (30).

Morawski J.M. 1980. *Methodological and physical bases of experimental model in sports.* Monografie nr 137, AWF Poznań (In Polish).

Nowikow A.A. i in. 1976. *O razrabotkie modielnych charaktieristik sportsmienow*. Tieorija i Praktika Fiziczeskoj Kultury nr 6.

Perl J. 1997. *Aspekte unkonventioneller Modellbildung – exemplarische Anwendungsmöglichkeiten in der Sportwissenschaft.* Zur Veröffentlichung angenommen bei: Sportwissenschaft.

Ryguła I., Wyderka Z. 1990. *Construction of optimization model of mechanical power development of 13-15-year old boys*. Wychowanie Fizyczne i Sport nr 2 (In Polish).

Ryguła I., Wyderka Z.1993. *Elements of control and optimization in sport training.* AWF Katowice (In Polish).

Ryguła I. 1999. *The calculating of optimal loads of exercise on the basis of the mathematical model*. Journal of Medical & Biological Engineering & Computing, vol 38.

Schlich W., Janssen J.P. 1990. *Der Einzelfall in der empirischen Forschung der Sportwissenschaft: Begründung und Demonstration zeitreihen-analytischer Methoden*. Sportwissenschaft, 3, 263-280.